



## COHERENT RECEPTION OF SIGNALS MODULATED BY PSEUDO-RANDOM SEQUENCES REFLECTED BY MOVING TARGETS

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**Abstract.** A novel method for coherent reception of the radar signals modulated by pseudo random sequence (PRS) is considered for detection of moving targets. The method completely suppresses time-delay (range) side-lobes, but increases the side-lobes in Doppler frequency axis. We consider how the Doppler shift of carrier frequency affects the efficiency of the novel method for coherent processing of the radar returns from a moving target. It was shown that as PRS with an increasing number of chips are applied, the side-lobe level indicators became comparable for both types of processing. If the side-lobe level along the Doppler frequency axis is of great importance for radar operation, then PRS with a large number of chips should be used. PRS with a small number of chips should not be used if the unambiguous detection and high velocity resolution is an important performance. However, in case of the Doppler frequency shift of the carrier frequency is not of fundamental importance, the proposed processing method can give a very good result, significantly exceeding that for the case of using conventional PRS processing: the fewer PRS chips, or the shorter PRS, the higher advantage we gain. In addition, the proposed method may be easily adapted for a digital implementation and, hence, is very easily amenable to a high degree of integration.

**Keywords:** Pseudo-Random Sequences, Cross-Correlation Function, Side-Lobes, Tailored Reference, Doppler Frequency Shift

### 1. Introduction.

In the paper [1] the authors suggested a novel method for coherent processing of signals modulated by pseudo-random sequences (PRS) in the radar receiver via estimation of cross-correlation of the radar returns and *tailored* reference and justified its efficiency in case of nonmoving targets. The method completely suppresses time-delay (range) side-lobes without decrease of the Signal-to-Noise Ratio (SNR) at the correlator output. However, PRSs, such as *m*-sequences, are very sensitive to the Doppler shift of the radar carrier frequency caused by a target motion, which requires especial consideration. In this paper we consider how the Doppler shift of carrier frequency affects the efficiency of the novel method for coherent processing of the radar returns from a *moving target* in the radar under consideration, suggested in [1].

### 2. Cross-correlation of *m*-sequence and tailored reference PRS signal

Auto-correlation function of the PRS,  $\mathbf{P}(\mathbf{t})$ , equals to [2, 3]:

$$\overline{\mathbf{P}(\mathbf{t}) \cdot \mathbf{P}(\mathbf{t} - \boldsymbol{\tau})} = \begin{cases} N; & \boldsymbol{\tau} = \mathbf{0} \\ -1; & \boldsymbol{\tau}_p < \boldsymbol{\tau} \leq T \end{cases} \quad (1a)$$



where  $\tau_p$  is the duration of PRS elementary pulse (chip);  $N = (2^k - 1)$  is the number of chips in the PRS period  $T$ , and  $k$  is a positive integer  $k \geq 2$ . Its average value is a constant for any instant of time:

$$\overline{P(t)} = -1. \tag{1b}$$

In [1] it was derived the following *tailored* reference sequence:

$$Y(t - \tau) = \frac{1}{2} [P(t - \tau) - 1], \tag{2}$$

which gives zero range side-lobes in the cross-correlation between PRS and the tailored reference sequence (2):

$$K(\tau) = \overline{P(t) \cdot Y(t - \tau)} = \begin{cases} \frac{N+1}{2} & ; \quad \tau = 0 \\ 0 & ; \quad \tau_p < \tau \leq T \end{cases} \tag{3}$$

It is seen that the *cross-correlation* function (3) has *zero* side-lobe level out of the zone of high correlation, i.e. for the delays,  $\tau$ , from the interval  $\tau_p < \tau \leq T$ .

### 3. Coherent Reception of Signals Modulated by Pseudo-Random Sequence Reflected by Moving Targets

The main performance of a radar based on signals with complex modulation may be estimated with the help of radar ambiguity function [4]. In the case of considering reflections from a moving target, usually, one introduces in the reflected signal additional modulation of its carrier frequency,  $f_0$ , due to Doppler effect. In this way, one has to obtain and investigate two-dimensional ambiguity function depending on the radar return delay,  $\tau$ , and Doppler frequency,  $\Omega$ :

$$\Omega \approx 4\pi f_0 \frac{v_0}{c},$$

where  $v_0$  is the target radial velocity and  $c$  is the speed of light.

Two-dimensional ambiguity function,  $\chi(\tau_i, \Omega)$  for a radar signal with PRS modulation is well known and may be presented as follows [5,6]:

$$\chi(\tau_i, \Omega) \equiv \chi(i, \Omega) = \frac{1}{N} \frac{\sin(\frac{\Omega \tau_p}{2})}{(\frac{\Omega \tau_p}{2})} \sum_{j=1}^N (P_{j+i}) P_j \cos[(j - 1)\Omega \tau_p] \tag{4}$$

where  $\tau_i = i\tau_p$  is overall delay time in the reference PRS;  $N$  is the number of PRS chips.

Summation in Eq.(4) is performed over all chips in the reference PRS.

To obtain cross-correlation function Eq.(3) between the *tailored* reference Eq.(2) and radar signal reflected by a moving target it is sufficient in Eq.(4) to substitute PRS in the round brackets  $(P_{j+i})$  by the *tailored* reference PRS  $(\frac{P_{j+i} - 1}{2})$ :



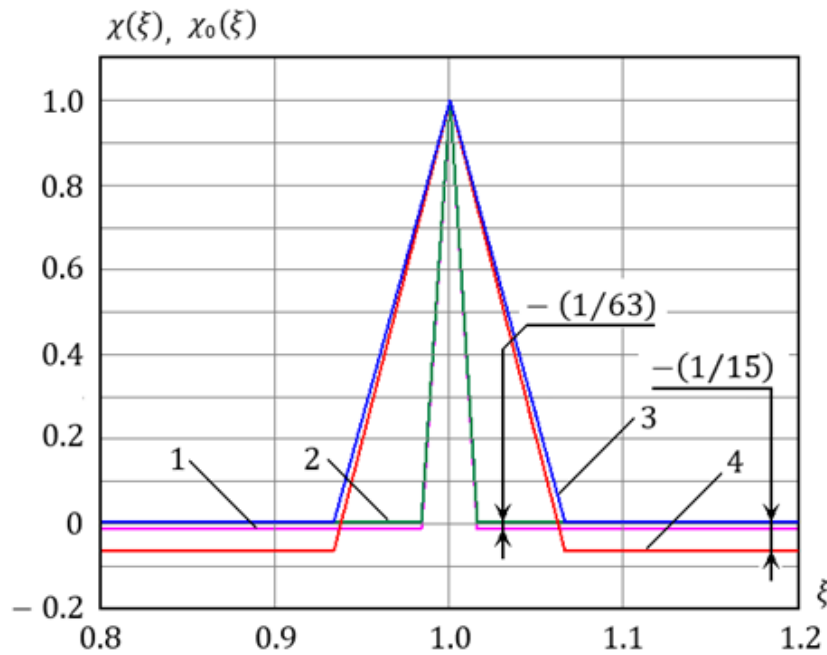
$$\chi_0(\tau_i, \Omega) \equiv \chi_0(i, \Omega) = \frac{2}{N+1} \frac{\sin(\frac{\Omega \tau_p}{2})}{(\frac{\Omega \tau_p}{2})} \sum_{j=1}^N \left(\frac{P_{j+i}-1}{2}\right) P_j \cos[(j-1)\Omega \tau_p] \quad (5)$$

where  $\tau_i = i\tau_p$  is overall delay time related to the  $i$ -th chip in the reference PRS;  $j$  is the number of the elementary pulse (chip) in the received PRS and in the tailored reference PRS Eq.(2).

Eq.(5) is written for the discrete delays  $\tau_i = i\tau_p$  with the step equals to chip duration  $\tau_p = T/N$ . We may simplify Eq.(5) if introduce dimensionless fractional delay  $\xi_i = \frac{\tau_i}{T} = i/N$  and the signal phase  $Z = (\Omega T)/N$  at the Doppler frequency  $\Omega$  acquired during one chip interval:

$$\chi_0(\xi_i, Z) = \frac{2}{N+1} \frac{\sin(Z/2)}{(Z/2)} \sum_{j=1}^N \left(\frac{P_{j+i}-1}{2}\right) P_j \cos[(j-1)Z] \quad (6)$$

Note that in our calculations, for demonstration of the cross-correlation properties,  $i$  may exceed the value  $N$  (Figure 1).



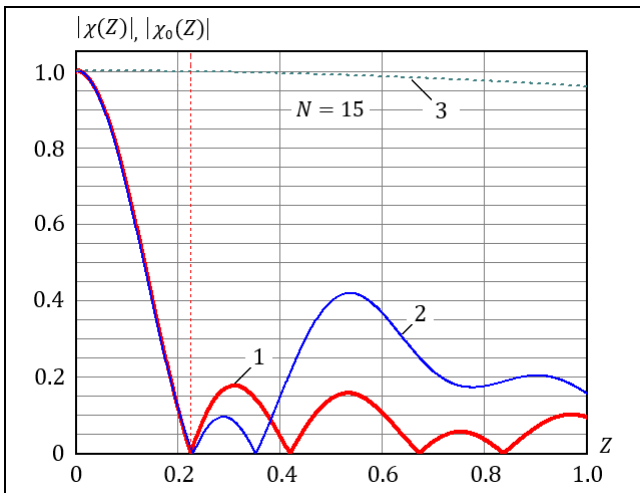
**Figure 1. Normalized cross-sections of ambiguity functions for PRS with N=63 and N=15, obtained with both conventional PRSs signal processing (curves 1 and 4, respectively) and proposed PRS signal processing (curves 2 and 3).**

In Figure 1, the cross-sections of the normalized per unit ambiguity function along the delays axis  $\xi_i$  are shown, being constructed according to Eq.(6) for PRS with the number of chips  $N=63$  and  $N=15$ , curves 2 and 3, respectively. The side-lobe level for conventional processing of PRS with  $N=63$  and  $N=15$  equal to  $(-1/63)$  and  $(-1/15)$ , respectively. In the same figure, for comparison, the normalized cross-section of the ambiguity function for PRSs obtained with conventional processing for the

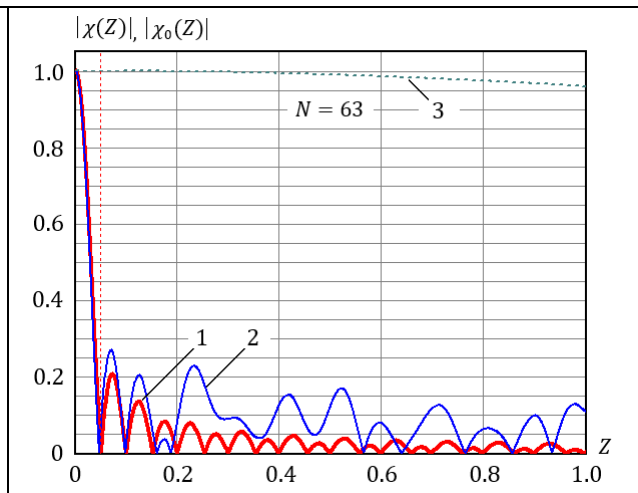


same number of PRS elementary pulses are shown as curves 1 and 4 for  $N=63$  and  $N=15$ , respectively. The Figure 1 clearly shows that in the latter case the side-lobe level is proportional to  $(-1/N)$ , while for the proposed processing method the side-lobe level equals zero for all time delays (curves 2 and 3).

Figure 2 and 3 show the modules of the cross-sections of the normalized per unit ambiguity functions  $\chi(\xi_i, Z)$  and  $\chi_0(\xi_i, Z)$  along the  $Z$ -axis proportional to the Doppler frequency ( $Z = (\Omega T)/N$ ), constructed according to Eq.(6) for the same PRS with the number of chips  $N=15$  and  $N=63$ , respectively. Curves 1 show ambiguity function  $\chi(\xi_i, Z)$  for conventional processing of PRS modulated signal, and curves 2 show the ambiguity functions  $\chi_0(\xi_i, Z)$  obtained with the suggested method for coherent processing of signals modulated by PRS (Eq.(6)). For comparison, the curve 3 shows the function  $\sin(Z/2)/(Z/2)$ , in the same figures.



**Figure 2. Cross-section of the ambiguity function along the  $Z$  axis, proportional to the Doppler frequency, for a PRS with the number of elements  $N=15$ :**  
**1 – for the usual processing of PRS;**  
**2 – for suggested processing of PRS;**  
**3– function  $\sin(Z/2)/(Z/2)$ .**



**Figure 3. Cross-section of the ambiguity function along the  $Z$  axis, proportional to the Doppler frequency, for a PRS with the number of elements  $N=63$ :**  
**1 – for the usual processing of PRS;**  
**2 – for suggested processing of PRS;**  
**3– function  $\sin(Z/2)/(Z/2)$ .**

It is evident from the above figures that the function  $\sin(Z/2)/(Z/2)$  has a very insignificant effect on the rate of side-lobe decay along the Doppler frequency axis. It is also evident that the side-lobe level for the suggested processing is noticeably higher than for the conventional PRS processing. But as PRS with an increasing number of chips are applied, the side-lobe level indicators became comparable for both types of processing (Figure 3).



## Conclusions

A method for coherent processing of the signals modulated by PRS (e.g. in the form of a periodic  $m$ -sequence of maximal length) is considered for detection of moving targets. In particular, this processing method will be very useful when long PRS signals cannot be used in radars, for example due to high target speeds, while the requirements for time-delay (range) side-lobes level are very high. It can be concluded that if the side-lobe level along the Doppler frequency axis is important for radar operations, then PRS with a comparatively large number of chips should be used. PRS with a small number of chips should not be used if the resolution along the Doppler frequency axis is an important radar performance. However, for radars operating in conditions where the Doppler frequency shift of the carrier frequency is not of fundamental importance, the proposed processing method can give very good results, significantly exceeding those for the case of using conventional PRS processing, in other words, when we use fewer PRS chips, or the shorter PRS, the higher advantage we gain. In addition, the proposed method may be easily adapted for a digital implementation and, hence, is very easily amenable to a high degree of integration.

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