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SUPPRESSION OF RADAR AMBIGUITY FUNCTION SIDELOBES USING MATRIX TRANSFORMATIONS

Volodymyr Konovalov
Leading Engineer
ORCID: 0009-0004-1932-4627
Lidiya Yurchenko
PhD, Senior Researcher
ORCID: 0009-0008-2608-7761
Institute for Radiophysics & Electronics NASU,
Kharkiv, 12 Ac. Proskura St., 61085

Abstract. *A method for cross-correlation processing of radar signals is proposed that allows one to achieve complete suppression of sidelobes of the ambiguity function along the time-delay axis at any length of a signal sample. The method is based on the properties of non-singular matrices allowing for exclusion of sidelobes by construction of a unit matrix for cross-correlation signal processing. The universality and simplicity of the method make it promising for implementation into modern radar systems with low probability of interception (LPI) for improving their detection capabilities. This opens new possibilities for the radar operation in complicated interference conditions and limited computational resources.*

Keywords: *ambiguity function, noise signals, sidelobes, cross-correlation processing, radar, matrix methods, interference suppression.*

Introduction

One of the main issues in the design of any radar system is the choice of the sounding signal. It determines the structure of the radar and the complexity of its construction, as well as its accuracy, noise immunity, and resolution.

The use of noise signals as radar signals is of great interest for developers of radars with high spatial resolution, low probability of interception (LPI) and low probability of exploiting (LPE) – signal information retrieval. There is a series of publications [1-9] studying the noise signals, developing noise radars for advanced applications, and comparing noise radars performance with that of conventional radars. Utilization of increasingly complex signals and advanced methods of their processing were proposed in [10-12] to improve radar performance.

In order to provide unambiguous range measurement and target detection, it is particularly important to obtain the radar Ambiguity Function (AF) with low levels of range and Doppler sidelobes, especially, for relatively short samples of radar signals.

In this paper, we propose a novel radar signal processing technique that ideally suppresses the sidelobe level down to zero for any sample length.

Research Methods and Results

It is known that, for a random signal with correlation (coherency) time much shorter then the signal duration and having a smooth shape of its power spectral density (PSD), the AF has a 'Thumbtack' shape provided long enough integration time [1-3,7]. In other words, the AF has no sidelobes in traditional understanding, which means that sidelobes may be averaged out for long enough integration time. For instance, if the signal PSD has a Gaussian shape, then the AF will also have Gaussian dependence along the time-delay (range) axis, which follows from Khinchin-Wiener theorem [1-3].

However, for the flatted (uniform) PSD within the signal frequency band, the AF shape is drastically transformed. Figures 1 and 2 shows the AF of random signal with flatted PSD and different values of the signal base $B = \Delta f \cdot T$ where Δf is the −3 dB (half-power) bandwidth of the signal power spectral density (PSD) and T is the signal duration. Both the level of sidelobes (LSL) and the main lobe width decrease with increasing the signal base B.

The dependence of the normal noise signal base B on the average level of sidelobes L_{SL} is evaluated to be as shown in Figure 3. As one can see, the LSL reduces to zero on both axes only in the limit of $B \to \infty$ where the AF becomes needle-shaped, i.e., a delta function. However, the longer is the realization of the noise signal (the signal duration T), the lower are the Doppler frequencies that allow unambiguous processing.

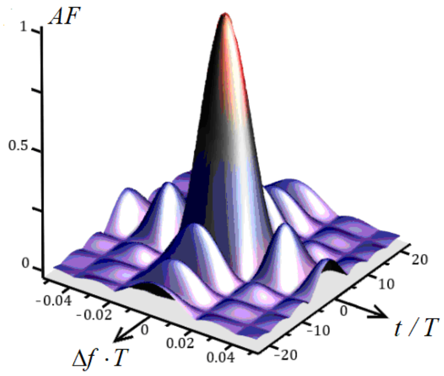


Figure 1 - The AF of the random signal at $B = 50$

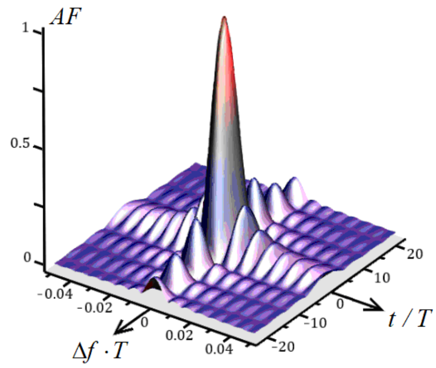


Figure 2 - The AF of the random signal at $B = 150$

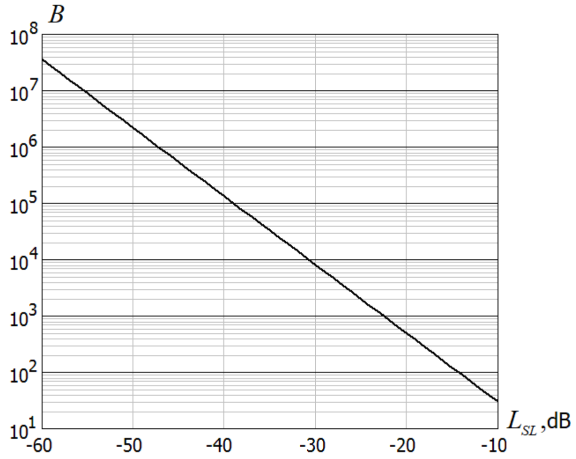


Figure 3 - Dependence of the normal noise signal base on the average sidelobe level

For example, for the sidelobe level on the time-delay axis to drop below -50dB, one needs a signal base of $B \ge 2 \cdot 10^6$. At the effective spectrum width of the noise signal $\Delta f = 500\text{MHz}$, which corresponds to the level of spatial resolution of 0.3 meters, the duration of the noise signal realization has to be $T = 4 \cdot 10^{-3}\text{s}$. It means that, with a signal of this duration, one can unambiguously determine the Doppler shift frequency $F_d \le 1/(2T) = 125\text{Hz}$ of the carrier frequency, which, for example, for a carrier wavelength of 8 mm corresponds to the target velocity of only $v_t \le F_d \cdot \lambda / 2 = 0.5\text{m/s}$ that is $v_t \le 1.8\text{km/hr}$.

This example shows that the method of reducing the LSL by increasing the duration of the noise signal realization has a limited range of applications. In this connection there is a necessity of such a method of noise signal processing, which would give a minimum LSL, preferably zero, at any duration of the noise signal realization.

The authors propose a universal method for suppressing the LSL of the radar AF along the time-delay axis almost to zero for an arbitrary noise or no-noise probing signal $c(t)$ of any duration provided that the signal distortion in the reflection process is small.

Let us make preliminary remark. Since the transfer of the modulating noise signal to the carrier frequency with subsequent demodulation does not affect the processing considered below, in order to simplify further expressions we will exclude the carrier from consideration, assuming that we emit and receive only the modulating noise signal, with which we will perform further transformations.

The proposed method is based on the fact that an arbitrary non-singular square matrix has the following property

$$\mathbf{A} \cdot \mathbf{A}^{\mathbf{-1}} = \mathbf{I} \qquad (1)$$

where $\mathbf{A} = [\mathbf{c}_1, \mathbf{c}_2 ... \mathbf{c}_n]$, $\mathbf{c}_j$ is the j-th column of the square matrix $\mathbf{A}$ with $n \times n$ elements, $\mathbf{A}^{\mathbf{-1}}$ is the inverse matrix, $\mathbf{I}$ is the unit matrix. Let us use this property to obtain a new reference signal.

First, we construct the matrix of the probing signal $\mathbf{A}$. For this purpose, the elements of the first column $\mathbf{c}_1$ of the matrix $\mathbf{A}$ are assigned to be the successive values of the probing signal sample of duration T_S at zero delay of this signal (the number of elements is $n \geq 2B$). In the second column $\mathbf{c}_2$ of the matrix $\mathbf{A}$ we record the sample of the noise signal obtained after the delay time $\Delta\tau$ where $\Delta\tau$ is the step of sampling of the noise signal ($\Delta\tau = T_S / (n-1)$ and $\Delta\tau \leq 1/(4\Delta f)$).

Thus, shifting the probing signal each time by one step of signal sampling, we will fill the whole matrix $\mathbf{A}$ (the whole signal duration is $T = 2T_S$). As a result, we obtain a matrix of the probing signal of total duration T with all possible values of the delay of the original signal $c(t)$ within the time interval T. If we neglect the distortion of the probing signal emitted to the target and returned to the processing device in the receiver, the received signal with proper matching of delay times will be one of the columns $\mathbf{c}_j$ of the matrix $\mathbf{A}$. Note that the matrix $\mathbf{A}$ is symmetric: its transposed matrix $\mathbf{A}^{\mathbf{T}}$ coincides with the original matrix, $\mathbf{A}^{\mathbf{T}} = \mathbf{A}$. The same is true for the inverse matrix: $\left(\mathbf{A}^{\mathbf{-1}}\right)^{\mathbf{T}} = \mathbf{A}^{\mathbf{-1}}$.

Let us now consider an arbitrary signal $r(t)$ at the receiver input represented by a discrete sample r_i of duration T_S in the form of a vector $\mathbf{r} = [r_1, r_2 ... r_n]$ where i is the index of the $r(t_i)$ signal value in the discrete sample recorded with the same time step $\Delta\tau$ as the probing signal (the input discrete signal is specified by the matrix $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2 ... \mathbf{r}_n]$). If the reference signal $b(t)$ with a discrete sample $\mathbf{b} = [b_1, b_2 ... b_n]$ is chosen to be the original probing signal $c(t)$, the correlator calculates the discrete cross-correlation function of the input and probing signals $k_j = \sum_{i=1}^{n} r_i b_{ij} = \sum_{i=1}^{n} r_i c_{ij}$ where $b_{ij} = c_{ij}$ are the elements of the matrix $\mathbf{A}$ (in matrix form $\mathbf{k} = \mathbf{rA}$ where $\mathbf{k} = [k_1, k_2 ... k_n]$). If the receiver input signal is the same as the probing signal $c(t)$, we obtain the autocorrelation function of the signal $q_j = \sum_{i=1}^{n} c_i c_{ij}$ (in matrix form $\mathbf{q} = \mathbf{cA}$) where the vector $\mathbf{c} = [c_1, c_2 ... c_n]$ is a discrete sample of the signal with zero delay. The discrete autocorrelation function of typical probing signals has sidelobes that need to be suppressed.

The method we propose consists in replacing the matrix of the probing signal $\mathbf{A}$, which is used for computing the correlation function $\mathbf{k}$, with the inverse matrix $\mathbf{B} = \mathbf{A}^{-1}$. That is, we choose the reference signal of the receiver to be defined by discrete samples, which form the columns of the inverse matrix $\mathbf{B} = \mathbf{A}^{-1}$. As a result, for the input signal $r(t) = c(t)$, cross-correlation function $\mathbf{k}$ computed at zero delay as the first row of the matrix product $\mathbf{K} = \mathbf{R} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{I}$ (or as the first column, due to the symmetry of matrices) has values $k_1 = K_{11} = 1$ at $j = 1$ and $k_j = K_{1j} = 1$ at $j > 1$. When the input signal delay corresponds to the k-th row of the delayed signals in the reference matrix $\mathbf{B}$, the output K_{kj} will be $k_j = K_{kj} = 1$ at $j = k$ and $k_j = K_{kj} = 0$ in all the other cases.

So, for any noise or no-noise probing signal $c(t)$ of any duration, undistorted during the propagation and reflection from the target and, therefore, generating the input signal $r(t) = c(t)$, we obtain a cross-correlation function with a single non-zero count at the coincidence of delays of the input and reference signals and zero value in other cases. In reality, at a certain distortion of the input signal, the cross-correlation function would have some non-zero lobes, which would be particularly informative against the background of their complete absence in the ideal case.

Thus, when computing cross-correlation matrix $\mathbf{K}$ of the input signal $r(t)$ given by matrix $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2 ... \mathbf{r}_n]$, the conventional method follows the definition of the reference matrix $\mathbf{B}$ as

$$\mathbf{B} = \mathbf{A} \quad \text{that results in} \quad \mathbf{K} = \mathbf{R} \cdot \mathbf{B} \quad \text{with sidelobes} \tag{2}$$

whereas the proposed method implements the assignment

$$\mathbf{B} = \mathbf{A}^{-1} \quad \text{that results in} \quad \mathbf{K} = \mathbf{R} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{I} \quad \text{with no sidelobes} \tag{3}$$

for all the signals, which differ from the probing signal only by time delays.

Discussion of the Results

Let us illustrate the results with a specific example. We will compose the input signal as the sum of two signals with different amplitudes. Let the amplitudes of the first signal reflected from a target closer to the radar and the second signal reflected from a more distant target to be 1 and 0.7 relative units, respectively. Then, the total signal is $\mathbf{r}_j^{(2)} = 1 \cdot \mathbf{c}_{j_1} + 0.7 \cdot \mathbf{c}_{j_2}$ where $\mathbf{c}_{j_1}$ and $\mathbf{c}_{j_2}$ are the discrete samples of the probing signal stored in different columns of the matrix $\mathbf{A}$, which correspond to different delay times or, what is the same, signal coming from different distances.

The effective bandwidth of the example signal is 20MHz, the duration is $3.4 \cdot 10^{-6}$ s, the base is $B = 68$. The maximum frequency of the Doppler signal that can be processed is $F_d = 1.5 \cdot 10^5$ Hz that corresponds to the maximum speed of the target 600m/s or Mach 1.8 at the carrier wavelength of 8mm. A signal of this duration can handle most cases encountered in practice.

The signals processed in this example are shown in Figure 4, (a) and (b), respectively.

Figures 5 and 6 show the autocorrelation and cross-correlation functions, respectively, for the signal $r^{(2)}$ reflected from two targets. Figure 5 shows the results of conventional processing whereas Figure 6 demonstrates complete suppression of sidelobes along the time axis using the proposed processing technique. As we can see, even so short signals can be processed using the proposed method and good results can be obtained.

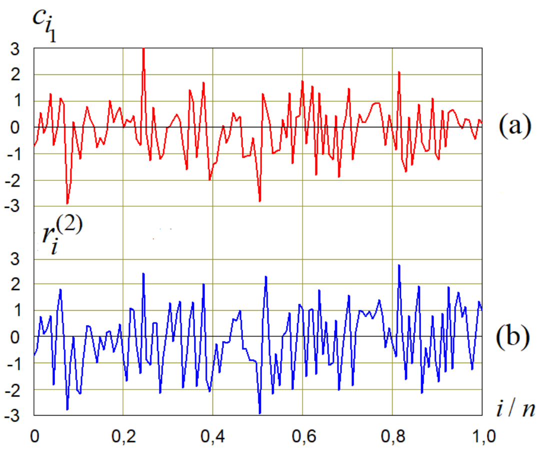
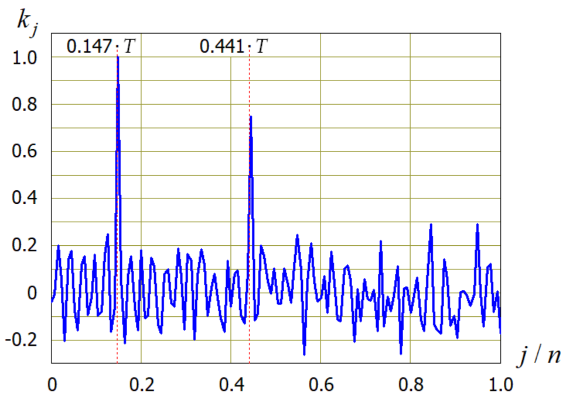


Figure 4 - Signal reflected from (a) closer target and (b) two targets

It should be noted, however, that generic sampling of a signal of duration T from the original noise ensemble may not necessarily allow one to construct an inverse matrix $\mathbf{B}=\mathbf{A}^{-1}$ since the original matrix $\mathbf{A}$ may not have a nonzero determinant. In this case, two simplest algorithms are possible [13]. The first one is the use of regularization, for example, the addition of a small identity matrix to matrix $\mathbf{A}$ and computing $\mathbf{B}$ as $\mathbf{B}=\left(\mathbf{A}+\varepsilon\mathbf{I}\right)^{-1}$ where ε is a small positive number.



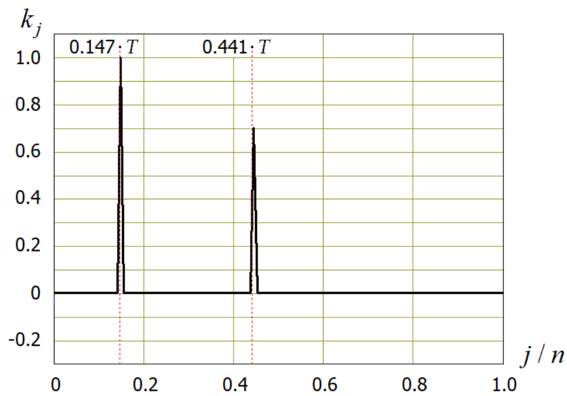


Figure 5 - Autocorrelation function of signal $r^{(2)}$ obtained with Eqs. (2)

Figure 6 - Cross-correlation function of signal $r^{(2)}$ obtained with Eqs. (3)

It is obvious that introducing a small additive into the matrix of reference signal would not change the properties of the signal itself. The additive can be chosen to be less than the receiver's noise. Then the additive would not play a significant role against the background of the receiver's own noise. This technique is often used in practice to "fix" singular matrices. This is the simplest way to fix a singular matrix from the point of view of practical implementation.

There is also a universal way of solving the problem. It consists in computing pseudo-inverse matrix $\mathbf{A}^{+}$ rather than the inverse one by applying singular value decomposition (SVD). Then everywhere in the formulas above we merely replace $\mathbf{A}^{-1}$ with $\mathbf{A}^{+}$. It is known that pseudo-inverse matrix exists for both singular matrices and rectangular ones. If $\det(\mathbf{A}) \neq 0$, the matrices $\mathbf{A}^{+}$ and $\mathbf{A}^{-1}$ are identical.

Conclusions

In this paper, a method of cross-correlation processing of noise signals is proposed, which allows for achieving complete suppression of the sidelobes of the ambiguity function along the time-delay axis for any length of signal sampling. The main results of the study include:

1. Theoretical significance.

The method is based on the unique properties of the inverse matrix, which allow for eliminating the influence of false sidelobes, preserving only the main maximum in the ambiguity function. This ensures accurate determination of the target range. The applicability of the method is confirmed for signals of various nature, including random and deterministic ones, which emphasizes its versatility and flexibility.

2. Practical applicability.

The method can be implemented in modern radar systems operating under the LPI and complex interference conditions. This opens up new opportunities for monitoring low-reflective targets in dense interference conditions. The elimination of the need of increasing the signal time base allows this method to be used in compact devices with limited computing resources.

3. Comparison with existing methods.

Unlike approaches that require significant increase in the signal duration to reduce the sidelobe level, the proposed method allows for achieving similar results without increasing the hardware complexity. Using regularization or singular value decomposition (SVD) allows one to bypass the limitation of singular matrices, improving the robustness of the method even under adverse conditions.

4. Extended application examples.

The ability to accurately determine target parameters such as range and velocity makes the method promising for radar detection of slow-moving targets, hidden objects, and low-level signals. The method is also applicable in radio communication for improving the quality of data transmission and reducing the probability of interception.

5. Potential development directions.

It is necessary to conduct additional studies of the influence of Doppler shift, interference and hardware errors on the efficiency of the method. This will allow the method to be adapted to real operating conditions. Developing adaptive filtering algorithms and improving temporal localization for dynamically changing signals can improve the accuracy of processing in complex multi-user systems. A promising direction is the implementation of the proposed method in integrated signal processing systems using artificial intelligence for the classification and analysis of radar data.

6. Evidence of effectiveness.

Experimental results have confirmed that the method achieves ideal sidelobe suppression even under low signal-to-noise ratio (SNR) conditions, making it particularly suitable for systems with high noise immunity requirements. Analysis of various types of signals showed a stable reduction in false detections, which increases the accuracy and reliability of radar and communication systems.

7. Summary and prospects.

The method presents a significant step forward in the field of radar signal processing, combining mathematical rigor and practical efficiency. Its versatility and wide application potential make the method promising for implementation in modern and future radar systems, including monitoring, surveillance, and air traffic control technologies.

In further research, it is planned to expand the proposed approach taking into account the influence of additional factors such as the Doppler shift of the carrier frequency, interference, and imperfections of the equipment itself, so that the method can be brought to practical application.

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