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## ANALYSIS OF THE LINEAR MACHINE WITH PERMANENT MAGNETS CHARACTERISTICS

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**Annotation.** In linear dynamic braking systems, the stator of the linear machine is made in the form of a multipole magnetic structure using high-coercivity permanent magnets, and the role of the rotor is performed by a ferromagnetic strip. However, such a system is characterized by low energy indicators, therefore, to increase them, a two-layer rotor is used, formed by a ferromagnetic strip clad with a highly conductive metal. Selection of the electrical conductivity and coating thickness of the combined strip allows you to optimize the braking process. At the same time, this approach does not provide a reduction in speed in the final section, which is necessary, for example, for free-fall towers. This is explained by the fact that with an increase in the coating thickness, the non-magnetic gap increases, which reduces magnetic induction, and therefore the interaction force at low speeds. Therefore, the purpose of this work is to develop a design solution for a combined strip that will provide a reduction in the speed of the vehicle in the final braking zone

**Key words:** linear motor, rotor, magnets, short-circuited winding, induction, speed of movement

### Introduction.

It is possible to reduce the non-magnetic gap by installing a combined strip in the end zone, made in the form of a massive toothed rotor of a linear machine.

The theory and calculation methods for machines of this design are still insufficiently developed. This is primarily due to the difficulties in determining the electrical parameters of the toothed zone. Existing calculation methods for machines



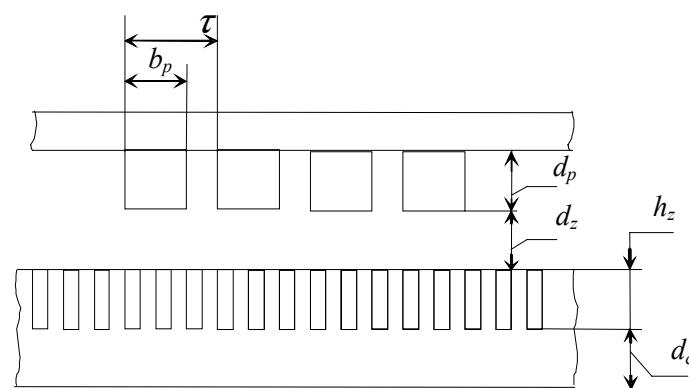
with a massive toothed rotor are based on traditional approaches using an equivalent circuit of the machine [2] or multilayer wave equivalent circuits of the active layer [3], the combination of which is carried out on the basis of the wave impedances of each layer, the values of which are determined by solving the field problem. The desire to obtain simple analytical relationships for determining the electrical parameters of equivalent circuits required the introduction of significant simplifying assumptions [2, 3] and an approximate solution of the equations of electrodynamics, which turned out to be of little use, especially for linear machines, taking into account the influence of edge effects.

### Analytical review.

Let's assume that permanent magnets of height  $\Omega$  with  $2p$  the number of poles and pole pitch  $\Omega$   $\tau$  are used to excite a spatially periodic magnetic field in a linear machine (Fig. 1). In the machine's calculation model, we extend the stator yoke along the  $d_p x$  -coordinate to  $\Omega_{\pm\infty}$ , and set its magnetic permeability to infinity [4].

The rotor is an infinitely long ferromagnetic strip with slots of size  $h_z \times b_n$ , in which squirrel cage rods of size are placed  $h_c \times b_c$ .

The tooth width  $b_z = t_z - b_n$  is the rotor  $t_z$  tooth pitch. Let the electrical conductivity of the cage rods  $\sigma_c$  and steel be  $\sigma_z$ . The width of the ferromagnetic strip is  $a$ , and the magnetic poles— $b$ .



**Figure 1 - Permanent magnets of a linear machine**



Thus, it is required to find solutions to the equations of electrodynamics

$$\begin{aligned} \text{rot}H &= j, \text{div}B = 0, \text{rot}E = -\partial B / \partial t, \\ B &= \mu H, j = \sigma [E + v \times B], \end{aligned} \quad (1)$$

in a multilayer medium with nonlinear and tensorial electrophysical parameters.

In the area of the rotor yoke ( $-d_c \leq z \leq 0$ ), following Neumann's suggestion [5], to take into account the nonlinear dependence of the magnetic permeability of steel, we set

$$\mu_c = \mu_e / (1 + z / z_c)^2 \quad (2)$$

where:  $\mu_e$  –magnetic permeability of steel at the boundary of the toothed zone ( $z = 0$ )

$z_c$  –the distance at which  $\mu_c$  it becomes equal to infinity and practically determines the depth of penetration of the magnetic field with nonlinear magnetic permeability of the medium.

Then from the system of equations (1) it is easy to derive the induction equation, which for  $Z$  –the component after applying the Fourier integral transform to it

$$B(n) = \int_{-\infty}^{\infty} B_z(x) e^{inx} dx \quad (3)$$

is reduced to the form:

$$(z_c + z)^2 \frac{\partial^2 B}{\partial z^2} + 2(z_c + z) \frac{\partial B}{\partial z} - \left( n^2 (z_c + z)^2 - i \mu_e \sigma_c z_c^2 n v \right) B = 0,$$

where:  $v$  –speed of movement of the strip in the direction of the axis  $x$ ,

$n$  –Fourier transform parameter.

The solution to this equation is

$$\begin{aligned} B &= C_1 (z_c + z) \frac{1}{2} \left[ I_v(n(z_c + z)) - \right. \\ &\quad \left. - \frac{I_v(n(z_c - d_c))}{K_v(n(z_c - d_c))} K_v(n(z_c + z)) \right] \end{aligned} \quad (4)$$

where:  $I_v(\dots), K_v(\dots)$  –modified Bessel functions of the first and second kind

$$v^2 = 1/4 - i \mu_e \sigma_c n v z_c^2,$$



$d_c$  –rotor yoke thickness.

The  $d_c > z_c$  second term in (4) should be set equal to zero.

Denoting at  $z = 0$

$$\left( \frac{\partial B}{\partial z} \right) / B = z_j \quad (5)$$

we arrive at mixed (impedance) boundary conditions for  $z = 0$ . From solution (4) it is easy to determine  $z_j$ , the approximate value of which for small arguments of the Bessel functions takes the form:

$$z_j \approx (\nu - 1) / z_c \quad (6)$$

To determine this,  $z_c$  we will use the parabolic approximation of the main magnetization curve of steel [6]:

$$B = kH^{1/m} \quad (7)$$

from which it follows using the condition  $\operatorname{div} B = 0$ :

$$\frac{\mu}{\mu_e} = \left( \frac{\partial B / \partial z}{\partial B / \partial z|_{z=0}} \right)^{\frac{1}{m}-1} = \frac{1}{(z + z_c)^2}$$

where  $m$  is the parabola index.

The last condition will be satisfied if

$$\operatorname{Re} \nu \approx 2m / (m - 1)$$

from where we find, according to (4):

$$z_c \approx \frac{2m}{m-1} d, \quad d = (2 / \mu_e \sigma_c |n v|)$$

and, accordingly:

$$z_j \approx \left( \frac{m+1}{2m} + i \right) / d \quad (8)$$

Since  $z_c$  and  $\nu$  are functions of  $\mu_e$ , which are not known in advance, solving the



problem will require the use of iterative methods, initially setting arbitrary values of  $\mu_e$ .

In the region of the toothed layer ( $0 < z < h_z$ ), due to the structural heterogeneity of the medium, in a two-dimensional formulation, the magnetic permeability has a tensor character:

$$\mu = \begin{vmatrix} \mu_x & 0 \\ 0 & \mu_z \end{vmatrix}$$

Then the equation for  $z$  the induction component after the Fourier transforms with respect to coordinate  $x$  (3) takes the form:

$$\frac{\partial^2 B}{\partial z^2} - \eta_1^2 B = 0, \quad \eta_1^2 = \frac{\mu_x}{\mu_z} \left( n^2 - i \mu_z \sigma n v \right) \quad (9)$$

where  $\sigma$  is the equivalent electrical conductivity of the tooth layer.

Its solution is the function

$$B = C_1 \operatorname{ch} \eta_1 z + C_2 \operatorname{sh} \eta_1 z \quad (10)$$

Satisfying the obtained solution to the conjugation condition - the equality of the normal components of the induction and the tangential components of the magnetic layer strength at the interfaces of the media  $z = 0$  and  $z = h_z$ , we find that in the air gap at  $z = h_z$  the impedance condition is satisfied:

$$\frac{\partial B}{\partial z} \Big|_{z=h_z} = z_0 \quad (11)$$

where

$$z_0 = \frac{\frac{\mu}{\mu_e} z_j + \frac{\mu}{\mu_x} \eta_1 \operatorname{th} \eta_1 h_z}{1 + \frac{\mu_x}{\mu_e \eta_1} z_j \cdot \operatorname{th} \eta_1 h_z}$$

Averaged over the tooth pitch length  $x$  -the component of magnetic permeability for a rectangular slot is equal to



$$\mu_x = \mu t_z / (b_n + \mu b_z / \mu_c) \quad (12)$$

Taking into account the effects of magnetic induction displacement in the massive tooth and rods of the cage, caused by eddy currents,  $z$  –the component of magnetic permeability averaged over the rotor tooth pitch is equal to:

$$\frac{\mu}{\mu_z} = \frac{t_z}{b_n h_z} \left\{ \frac{\frac{h_z - h_c}{2th(\lambda_z b_z/2)} \frac{\mu_e}{\mu} \frac{b_z}{b_n} + \frac{h_c}{2th(\lambda_c b_c/2)} \frac{\mu_e}{\mu} \frac{b_c}{b_n} + 1 - \frac{b_c}{b_n}}{\frac{\lambda_z b_z}{\lambda_c b_c} + 1} \right\} \quad (13)$$

where:  $\lambda_c = (i \mu \sigma_c |n v|)^{0,5}$  –the depth of penetration of the magnetic field into the rods of the squirrel cage,

$\lambda_z = (i \mu_e \sigma_z |n v|)^{0,5}$  –the penetration depth of the magnetic field in the massive teeth of the rotor.

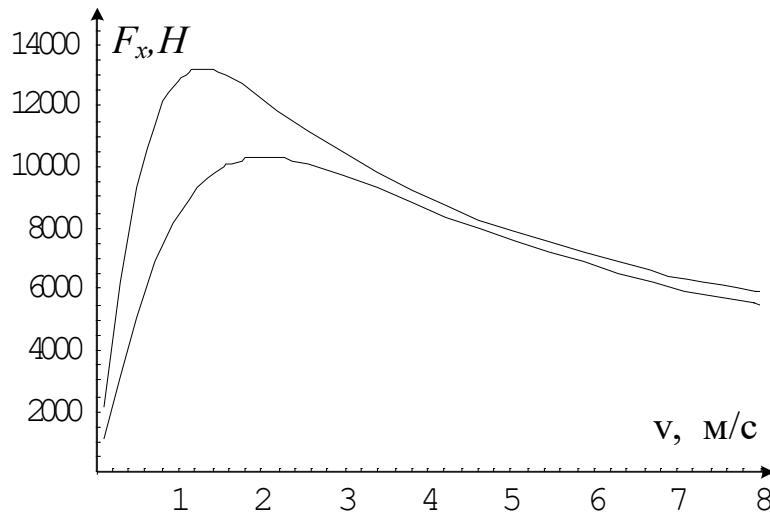
The magnetic permeability component  $\mu_z$  is complex, driven by the influence of eddy currents in the rotor's massive teeth, leading to a phase shift in magnetic induction relative to the magnetic field strength. This accounts for power losses due to eddy currents in the teeth and rods of the squirrel cage.

A comparison of the mechanical characteristics of a linear machine with a squirrel cage in a ferromagnetic strip and a combined busbar with a copper coating is shown in Fig. 2.

Here the upper curve describes the electrodynamic forces depending on the speed of the rotor with a squirrel cage, and the lower one - with a metal coating [1]. The comparison was based on the same design of both machines, in which the cross-section of the secondary squirrel cage was assumed to be equal to the cross-section of the metal coating for the rotor design with a slot width  $b_n$ , equal to the thickness of the cell rod  $b_c$  and, accordingly, the thickness of the metal coating  $d_m$ . The height of the rod  $h_c$  is equal to the height of the groove  $h_z$  and is equal to the tooth pitch  $t_z$  rotor. The thickness



of the rotor yoke was assumed to be the same in both cases.



**Figure 2 - Mechanical characteristics of a machine with a coated ferromagnetic rotor and with squirrel cage**

As can be seen, the advantage of a squirrel cage rotor is observed only at low rotor speeds, and they become virtually equivalent at higher speeds, in this example at  $v > 3 \text{ m/s}$ . Therefore, a squirrel cage rotor should only be used in the final braking zone, which allows for a reduction in speed in the residual braking zone by almost half.

**Conclusions.** A method for calculating the electromagnetic processes and mechanical characteristics of a linear machine with permanent magnets and a solid-toothed squirrel-cage rotor has been developed, taking into account the nonlinear magnetic permeability of steel. The use of ferromagnetic teeth allows for an increase in the magnetic field in the gap and electrodynamic forces only at low rotor speeds.

The equivalent electrical conductivity of the tooth layer is determined by the sum of the electrical conductivities of the squirrel cage rod and the massive ferromagnetic tooth.

In the presence of a squirrel cage, the influence of conductive steel teeth has little effect on the mechanical characteristics of the machine, but allows braking speed in the end zone to be reduced by half.

The end zone of the combined tire of the "free fall tower" attractions should be made using a squirrel cage.



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