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COMPARATIVE ANALYSIS OF ANALYTICAL CALCULATIONS AND COMPUTER SIMULATION OF TRANSIENT PROCESSES IN LINEAR ELECTRICAL CIRCUITS

Savolova E.V. / Савьолова Е.В.
senior lecturer / ст. викладач.
ORCID: 0000-0001-9266-9323
Odesa Polytechnic National University, Odesa, Shevchenka ave. 1, 65045
Національний університет «Одеська політехніка», Одеса, пр-кт Шевченка 1, 65045

Abstract. *The article studies transient processes in linear electrical circuits excited by piecewise linear voltage sources. Analytical calculations performed on the basis of the Duhamel convolution integral are compared with the results of computer simulation performed in the Multisim Live cloud environment. A first-order electrical circuit is considered as an example. The study confirms the effectiveness of simulation modelling as a practical alternative to analytical calculations, providing visual visualization, interactive measurement tools, and instant feedback. The combination of theoretical methods and computer simulation ensures both accuracy and practical applicability of the results, which is especially important for engineering practice and educational purposes.*

Key words: *transient processes, linear electrical circuits, Duhamel integral, piecewise linear voltage, simulation, Multisim Live.*

Introduction.

The investigation of transient processes is one of the principal tasks of both theoretical and applied electrical engineering, since during their occurrence circuits may experience dangerously high voltages or currents, which directly affect the dynamic properties, reliability, and energy efficiency of electronic and electrotechnical systems.

In practical electrical engineering, voltages of complex waveform are frequently employed in systems such as:

- control of power electronics to achieve smooth start-up or braking, for example, in soft-start motor circuits;
- regulation of position, speed, or current through the generation of triangular or trapezoidal signal forms for sweep control in oscilloscopes or modulation in PWM controllers;
- shaping of motion profiles in CNC machines and industrial robots, ensuring minimal overload and high positioning accuracy;
- electric drives and automation systems to reduce dynamic losses and enhance

operational stability…

One particular type of such signals is the piecewise linear voltage—a sequence of distinct time intervals governed by different, yet simple, linear laws of variation.

For calculating transient processes in circuits with such signals, both classical and operational methods may be applied, especially when the voltage assumes trapezoidal, triangular, or rectangular forms. However, in cases where a linear electrical circuit is subjected to a voltage that varies according to a complex time law and contains discontinuities of the first kind, the analysis of the transient process is typically carried out using the Duhamel integral [1, 2, 3, 4].

Overview of the external environment.

A survey of the literature demonstrates that the Duhamel integral remains a widely demanded method of calculation not only for linear electrical circuits. Examples of its application include the analysis of electromagnetic processes in nonhomogeneous media (such as lightning discharges and industrial disturbances), practical use in control systems of integer and fractional order [5], in computer-based measurement systems, and for determining Bode and Nyquist characteristics of automation objects [6].

Among the disadvantages of the Duhamel integral are the requirement of zero initial conditions and the inability to account for switching operations that alter the structure of the electrical circuit (schemes). However, in works [7, 8, 9] the possibility of overcoming these limitations has been substantiated through the use of discontinuous functions, and solutions have been proposed for the problem of calculating transient processes in electrical circuits with nonzero and “incorrect” initial conditions.

Input and Methods.

Well-known analytical methods share a common drawback: the complexity of calculations increases significantly with the order of the circuit and the number of voltage intervals governed by different laws of variation. This leads to cumbersome expressions, raises the probability of computational errors, and complicates the analysis.

One of the approaches that substantially simplifies the study of such processes is simulation modelling [10].

The purpose of this work is to demonstrate the advantages of modelling transient processes in circuits with piecewise linear voltage using the cloud-based environment Multisim Live [11], as well as to compare the obtained results with analytical calculation methods.

The study has been carried out on the example of a simple first-order circuit with the following element parameters: R_1 = 18 Ω, R_2 = 35 Ω, R_3 = 25 Ω, L_1 = 50 mH. The circuit is connected to a source with piecewise linear voltage (Fig. 1). The Value tab in the properties window and the graphical representation of the voltage variation law according to the specified parameters are shown in Fig. 2.

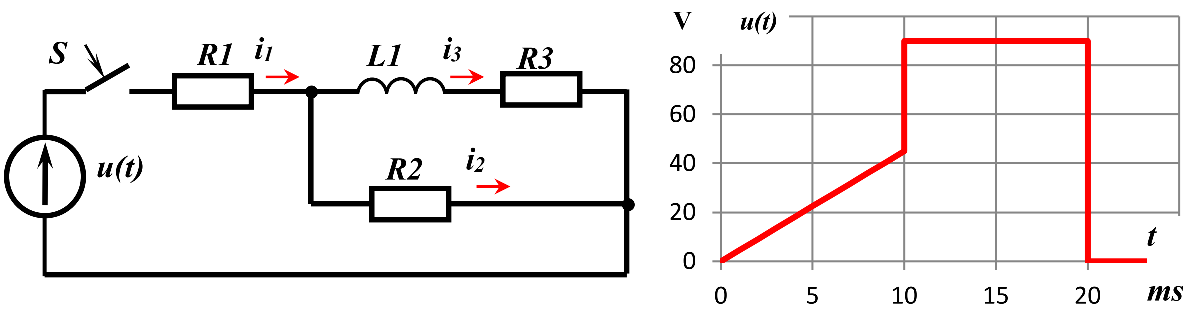
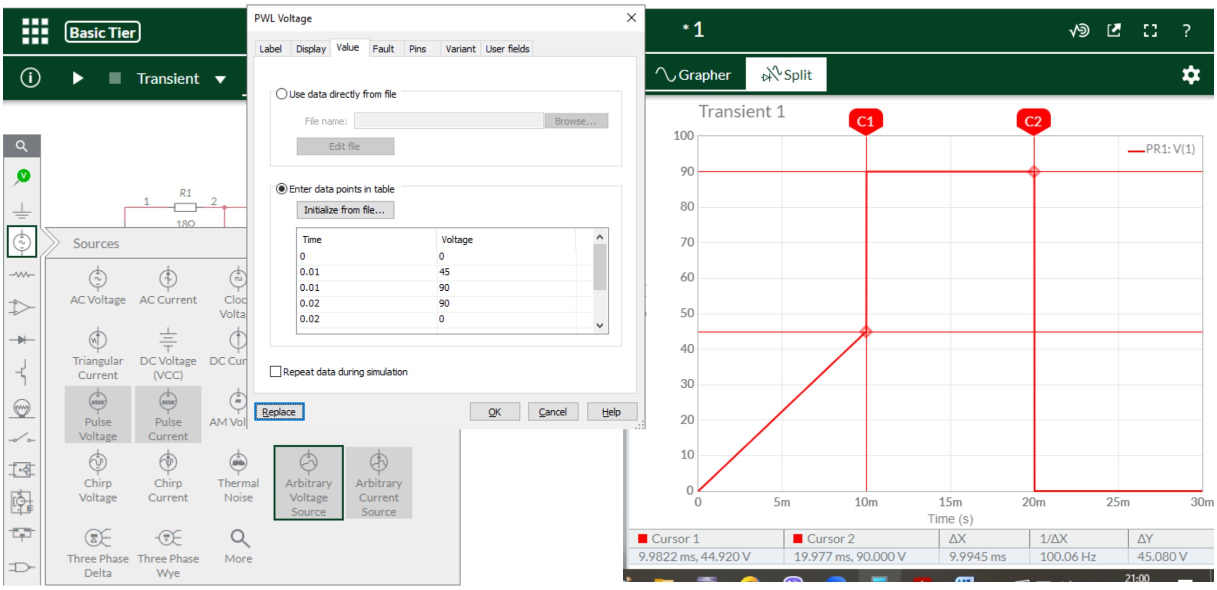


Figure 1 – Schematic diagram and graph of voltage change

Authoring



○ Use data directly from file

● Enter data points in table

Time	Voltage
0	0
0.01	45
0.01	90
0.02	90
0.02	0

☐ Repeat data during simulation

Cursor 1	Cursor 2	ΔX	1/ΔX	ΔY
9.9822 ms, 44.920 V	19.977 ms, 90.000 V	9.9945 ms	100.06 Hz	45.080 V

Figure 2 – Parameters of the piecewise linear voltage source

Authoring

The law of current variation in resistor *R₂* was obtained in the Transient simulation mode. To observe the results, probes were employed, providing graphical representation of the voltage at the circuit input (Fig. 2) and the current through the resistor $i_{R_2}(t)$.

Research results.

To verify the correctness of the simulation results, the law of current variation $i_{R_2}(t)$ was calculated using the Duhamel integral. According to the algorithm, the transient conductance $g(t)$ for the desired current was determined. Its expression corresponds to the law of current variation in the resistor when the circuit is connected to a unit voltage source.

Considering the zero initial condition $i_{L_1}(0_+) = i_{L_1}(0_-) = 0\,A$ the forced component of the current is given by:

$$i_{R_2}(t)_f = \frac{E}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}} \cdot \frac{R_3}{R_2 + R_3} = \frac{1}{18 + \frac{35 \cdot 25}{35+25}} \cdot \frac{25}{35 + 25} = 12.79\,mA.$$

The characteristic equation:

$$\frac{R_1 \cdot R_2}{R_1 + R_2} + R_3 + pL_1 = 0$$

$$\frac{18 \cdot 35}{18 + 35} + 25 + 0.05p = 0$$

$$p = -\frac{36.87}{0.05} \approx -737.74\,s^{-1}.$$

The integration constant A -

$$A = i_{R_2}(0) - i_{R_2}(0)_{\mathrm{y}} = 18.87 - 12.79 = 6.08\,mA,$$

where

$$i_{R_2}(0) = \frac{E}{R_1 + R_2} = \frac{1}{18 + 35} = 18.87\,mA.$$

Thus, the law of current variation is:

$$i_{R_2}(t) = i_{R_2}(t)_f + i_{R_2}(t)_n = 12.79 + 6.08e^{-738t}, mA.$$

The corresponding expression for conductance:

$$g(t) = 12.79 + 6.08e^{-738t}, mS.$$

The laws of input voltage variation were analysed for each time interval.

Interval 1 ($0 \leq t < t_1$):

- voltage law

$$u_1(t) = \frac{U_1 - U_0}{t_1} * t = \frac{45 - 0}{0{,}01} * t = 4500t, A;$$

- derivative

$$u_1'(\tau) = 4500\,\frac{V}{s};$$

- voltage jump

$$U_{01} = 0\,V.$$

Interval 2 ($t_1 \leq t < t_2$):

- voltage law

$$u_2(t) = U_2 = 90\,V;$$

- derivative

$$u_2'(\tau) = 0, \frac{V}{s};$$

- voltage jump

$$U_{02} = U_2 - U_1 = 90 - 45 = 45\,V.$$

Interval 3 ($t_2 < t \leq \infty$):

- voltage law

$$u_3(t) = 0, V;$$

- derivative

$$u_3'(\tau) = 0, \frac{V}{s};$$

- voltage jump

$$U_{03} = u_3(t_2) - U_2 = 0 - 90 = -90\,V.$$

This enables the correct application of the Duhamel integral for further calculation of the current in the circuit,

beginning with the first interval

$$i_2(t) = U_{01} \cdot g(t) + \int_0^t u_1'(\tau) g(t - \tau) d\tau =$$

$$= 0 \cdot (12.79 + 6.08e^{-738t}) \cdot 10^{-3} + \int_{0}^{t} 4500 \cdot \left(12.79 + 6.08e^{-738(t-\tau)}\right) \cdot 10^{-3}\, d\tau$$

$$= 57.555t\big|_0^t + \frac{27.36}{738} e^{-738t} e^{738\tau}\Big|_0^t = 0.037 + 57.555t - 0.037e^{-738t}, \text{A};$$

- for the second interval

$$i_2(t) = U_{01} \cdot g(t) + \int_{0}^{t_1} u_1'(\tau) g(t-\tau) d\tau + U_{02} \cdot g(t - t_1) + \int_{t_1}^{t} u_2'(\tau) g(t-\tau) d\tau$$

$$= 0 \cdot (12.79 + 6.08e^{-738t}) \cdot 10^{-3} + \int_{0}^{t_1} 4500 \cdot \left(12.79 + 6.08e^{-738(t-\tau)}\right) \cdot 10^{-3}\, d\tau$$

$$+ 45 \cdot \left(12.79 + 6.08e^{-738(t-t_1)}\right) \cdot 10^{-3} + \int_{t_1}^{t} 0\, d\tau =$$

$$= 57.555 \cdot t\big|_0^{t_1} + 0.037e^{-738t} e^{738\tau}\big|_0^{t_1} + 0.57555 + 0.2736e^{-738(t-t_1)}$$

$$= 0.57555 + 0.037e^{-738(t-t_1)} - 0.037e^{-738t} + 0.57555 + 0.2736e^{-738(t-t_1)} =$$

$$= 1.1511 - 0.037e^{-738t} + 0.3106e^{-738(t-t_1)}, A;$$

- for the third interval

$$i_2(t) = U_{01} g(t) + \int_{0}^{t_1} u_1'(\tau) g(t-\tau) d\tau + U_{02} g(t - t_1) + \int_{t_1}^{t_2} u_2'(\tau) g(t-\tau) d\tau$$

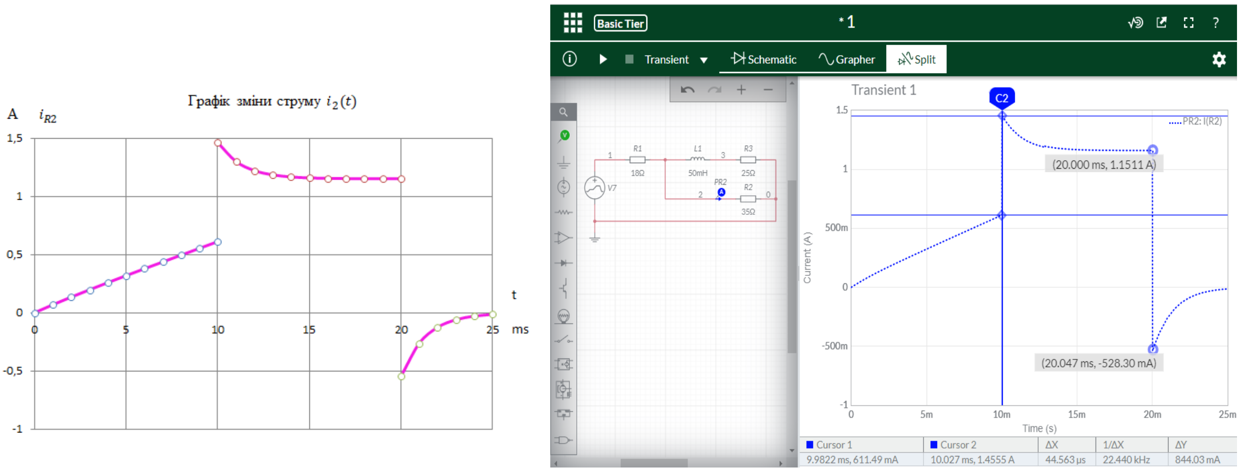
$$+ U_{03} g(t - t_2) = \int_{0}^{t_1} 4500 \cdot \left(12.79 + 6.08e^{-738(t-\tau)}\right) \cdot 10^{-3}\, d\tau +$$

$$+45 \cdot \left(12.79 + 6.08e^{-738(t-t_1)}\right) \cdot 10^{-3} - 90\left(12.79 + 6.08e^{-738(t-t_2)}\right) \cdot 10^{-3} =$$

$$= 1.1511 - 0.037e^{-738t} + 0.3106e^{-738(t-t_1)} - 1.1511 - 0.5472e^{-738(t-t_2)} =$$

$$= -0.037e^{-738t} + 0.3106e^{-738(t-t_1)} - 0.5472e^{-738(t-t_2)}, A.$$

The graph constructed in Microsoft Excel based on the results of the analytical calculation, together with the simulation results obtained in Multisim Live, are presented in Fig. 3.



Cursor 1	Cursor 2	ΔX	1/ΔX	ΔY
9.9822 ms, 611.49 mA	10.027 ms, 1.4555 A	44.563 μs	22.440 kHz	844.03 mA

Figure 3 – Graf of the current $i_2(t)$ in Microsoft Excel and simulation result in the Multisim Live environment

Authoring

Discussion and analysis of results.

The analytical results were independently verified through an alternative derivation using an artificial intelligence - based computational tool [12].

For ease of comparison, the results of calculation and simulation are presented on the same scale. On the oscilloscope trace, cursors are placed to indicate the values of the investigated quantity.

The comparison of analytical calculations and computer simulation demonstrates a high degree of consistency, confirming the reliability of both approaches.

This agreement highlights the robustness of simulation modelling as a practical alternative to analytical calculations, providing intuitive visualization, interactive measurement tools, and immediate feedback - features that are particularly valuable for educational purposes and engineering practice.

Closing and conclusions.

The comparative analysis of calculations and simulation modelling of transient processes in a first-order circuit with piecewise-linear voltage makes it possible to draw the following conclusions:

- analytical calculation using the Duhamel integral is characterized by high computational complexity and a significant probability of errors as the circuit order and the number of intervals increase; it requires thorough mastery of the mathematical apparatus and great attentiveness when working with cumbersome formulas;

- simulation modelling in the cloud environment Multisim Live greatly simplifies the process of obtaining results, ensures clarity and convenience of analysis, and minimizes the occurrence of computational errors;
- the coincidence of graphical results confirmed the correctness of the modelling.

In summary, computer modelling in the cloud environment Multisim Live indeed allows for a significant simplification of the study of transient processes. It can be applied in the analysis of complex electrical circuits as an accessible and intuitive tool for learning, research, and verification of calculations, especially in cases involving complex forms of excitation signals.

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