http://www.moderntechno.de/index.php/meit/article/view/meit42-02-044
DOI: 10.30890/2567-5273.2025-42-02-044

UDC 699.88

PROPAGATION OF LOCAL HEAT FLUXES IN LAMINATED MEDIA

Pysarenko A.M.
c.ph.-m.s., as.prof.
ORCID: 0000-0001-5938-4107
Odessa State Academy of Civil Engineering and Architecture,
Odessa, Didrihsona, 4, 65029

Abstract. *This work examines the kinetics of local heat flux propagation in multilayer laminate structures in cases where one or more inner layers possess sufficiently high thermal conductivity to dissipate heat laterally under non-uniform heating conditions. The computational model is implemented in two stages, which respectively describe two regions of interest: the heated region and the fin (cooling) region. This model predicts the behavior of laminate films when heated by a constant temperature or heat flux source acting on only part of the surface. The one-dimensional, steady-state, two-region fin model is unique in that it describes conductive heat transfer exclusively in the heat-spreading layer for both Cartesian and cylindrical coordinates. Complete temperature uniformity is achievable with applied heat fluxes. At the same time, when using constant temperature sources (e.g., phase change materials), laminates provide higher heat transfer rates compared to traditional homogeneous encapsulation materials. The two-region fin model allows for the calculation of temperature profiles and heat transfer rates for steady-state or pseudo-steady-state analysis, which is validated by the results of two-dimensional finite element modeling under applicable conditions.*

Key words: *multilayer laminate, heat spreading, thermal conductivity, fin model, heat transfer, steady-state analysis.*

Introduction.

The study of heat propagation in multilayered composite structures is a critical area of modern materials science and thermal engineering, driven by the increasing demand for efficient thermal management in microelectronics, aerospace components, and energy storage systems. Laminated composites, characterized by alternating layers of materials with distinct physical properties, offer unique opportunities to tailor thermal responses that are unattainable with monolithic materials. This introduction explores the kinetics of both local and non-local heat flux propagation within these structures, focusing on how internal architecture influences macroscopic thermal behavior. A primary challenge in analyzing these systems lies in the disparate scales of the layers and the interfaces between them, which necessitate specialized computational approaches depending on the complexity of the laminate stack and the presence of foreign inclusions.

Local heat flux propagation typically refers to situations where the thermal gradient is confined to a specific area, such as a localized hotspot on a circuit board or

a point of impact on a protective shield. In such scenarios, the ability of internal layers to spread heat laterally becomes the defining factor for the structural integrity and performance of the device. High-thermal-conductivity layers embedded within a low-conductivity matrix act as thermal bridges, redistributing energy away from the source to prevent localized overheating. This lateral spreading is highly sensitive to the thickness of the layers and the quality of the thermal contact at the interfaces. When the heating conditions are non-uniform, the internal layers must possess sufficient conductivity to carry heat across the domain, a process that is effectively described by regional models that distinguish between the zone of active heating and the surrounding cooling or fin regions.

Non-local heat flux propagation introduces further complexity, particularly when the mean free path of heat carriers becomes comparable to the characteristic dimensions of the layers [1]. In these instances, traditional Fourier-based models may fail to accurately predict temperature profiles, as the thermal energy does not diffuse in a simple gradient-driven manner but instead exhibits ballistic or quasi-ballistic behavior. This is especially relevant in modern thin-film laminates where layer thicknesses are reduced to the nanometer scale. The interaction of heat fluxes across multiple boundaries requires an understanding of how energy is partitioned and dissipated throughout the entire composite volume, rather than just in the immediate vicinity of the heat source.

The existing computational methodologies for analyzing these structures are generally categorized based on the number of laminar layers and the geometric complexity of the system. For cases involving a small number of layers, discrete modeling techniques are often employed. These methods treat each layer as a separate entity with its own governing equations, allowing for a precise determination of the temperature field across each interface. Such models are highly accurate for predicting the performance of simple bimaterial laminates or trilayers used in experimental setups. They provide a detailed view of the thermal resistance at each boundary, which is crucial for applications where the interface itself constitutes a significant portion of the total thermal impedance.

However, as the number of layers increases, the computational cost of discrete modeling becomes prohibitive. For structures with a large number of laminar layers, homogenization techniques or effective medium theories are preferred. These methodologies treat the multilayered stack as a single anisotropic continuum with effective thermal conductivity tensors. This simplification allows for the analysis of complex, bulk-scale composite parts while maintaining the essential physics of the directional heat flow. The transition from discrete to homogenized models represents a shift from focusing on individual interfacial events to a more global view of thermal diffusion. These bulk models are particularly useful in the design of large-scale structural composites where the primary goal is to maintain a stable global temperature profile under varying environmental loads.

A third category of interest involves cases with a low concentration of foreign inclusions of varying geometries within a laminated matrix. These inclusions might be reinforcing fibers, carbon nanotubes, or metallic particles added to enhance specific mechanical or thermal properties. The presence of these elements introduces local perturbations in the heat flux lines. When the concentration is low, the interaction between individual inclusions can often be neglected, allowing for the use of perturbation methods or dilute limit approximations. The geometry of the inclusion that is whether spherical, cylindrical, or planar that is drastically alters the local thermal environment. For example, high-aspect-ratio inclusions like fibers are exceptionally effective at directing heat along their longitudinal axis, creating preferential pathways for energy dissipation.

In the context of laminated structures, these inclusions often reside at the interfaces or within specific "buffer" layers. The modeling of such systems requires a hybrid approach that combines the principles of laminate theory with the physics of composite inclusions. For small concentrations, the focus is on how a single particle or a small cluster of particles alters the effective conductivity of a single layer, which is then integrated back into the global laminate model. This multi-scale perspective is essential for developing "smart" materials where the thermal path can be intentionally manipulated through the strategic placement of heterogeneous elements.

The validation of these various models that is from the two-region fin model for lateral spreading to the homogenized models for thick stacks that is typically achieved through comparison with multi-dimensional finite element analysis. These numerical simulations provide a high-fidelity reference for the steady-state and pseudo-steady-state temperature profiles predicted by simpler analytical frameworks. By comparing the rate of heat transfer and the uniformity of temperature distribution across different modeling scales, researchers can define the limits of applicability for each method.

Ultimately, the goal of studying heat flux propagation in these complex systems is to achieve a level of thermal control that surpasses that of conventional materials. Laminated structures provide a platform for achieving near-perfect temperature uniformity under applied fluxes or significantly enhanced heat transfer rates when coupled with phase change materials or active cooling sources. Whether dealing with a simple three-layer film or a laminated composite with thousands of plies and micro-inclusions, the underlying principles of lateral spreading and effective medium response remain the cornerstones of thermal design [2]. This work seeks to bridge the gap between these different regimes, providing a comprehensive overview of how heat moves through the structured interfaces of modern composite materials.

Multi-zone model of local heat flows on two-dimensional structures

The basic assumption of the computational methodology is the independence of the thermal conductivity and external heat transfer coefficients from the distance to the boundary surfaces of the laminated composite sample. Furthermore, the magnitude of the thermal expansion of the materials can be neglected. The multi-region fin problem reduces a two-dimensional system into a set of one-dimensional symmetric steady-state heat transfer equations solved only for the inner high thermal conductivity metal core, but in two parts: the heated region and the lateral surface region. Solutions were obtained for both Cartesian and cylindrical coordinate systems. Although the scope of application for this multi-region model is quite broad, consideration must be given to the implications of the core assumptions to justify the validity of such an approach. The heat spreading process can be represented as the dominance of transverse heat conduction (x or r directions) over planar heat conduction (z-direction) in the high

thermal conductivity metal core. These assumptions amount to the statement that the planar thermal resistance dominates over the transverse resistance. In terms of an order of magnitude estimation, these relationships are as follows:

$$R = d_1/k_1 + d_2/k_2 + \ldots + 1/h\,, \quad (1)$$

where

$d_1, d_2, \ldots$ – thicknesses,

h - convection heat transfer coefficient,

k_1, k_2 - thermal conductivity.

It should be noted that the multi-region model of local heat flux propagation can also be used for pseudo-steady-state analysis of laminated systems in cases of both small and large numbers of laminar layers. Laminated structures are typically thin, so pseudo-steady-state analysis can often be considered. This condition is characterized by the fact that the time derivative of the heated length is sufficiently small compared to the heat accumulation capacity of the laminate in the presence of local foreign inclusions located approximately midway between two adjacent laminar layers. The assumption is that the bottom thermally resistive layer in the fin region can be treated as well-insulated. However, a sufficient number of composite materials possess low thermal conductivity, which results in only minimal self-heating of the material beneath the lateral surface region. The latter does not constitute a parasitic heat loss either, since the heat conduction path is internal to the system, and the heat would be available for heat transfer with the environment nonetheless. The preferred path for conduction heat transfer in the lateral surface region would remain the same as in the ideal case just presented.

The Cartesian thermal energy balance equation applied exclusively to the high thermal conductivity metal core in the heated region with a constant applied temperature (T_i) is as follows

$$\frac{d^2T}{dx^2} - \frac{k_2}{d_1 d_2 k_1}(T - T_i)^2 - \frac{1}{R_1 d_1 k_1}(T - T_i) = 0, \quad (2)$$

where

$R \sim h^{-1}$.

In this study of the two-region fin problem, two aspects of heat transfer remain to be considered: the overall linear power density entering and leaving the system, and the linear power density for the lateral surface only, which crosses the boundary of the heated region. A characteristic parameter of the calculation methodology is the ratio of the heat transfer rate in the laminate to the heat transfer rate in a simple encapsulation made of the same durable but low-thermal-conductivity outer layers of the laminate and having the same overall thickness. It is assumed that the simple encapsulation allows only planar heat transfer. Since the conditions ensuring the accuracy of the basic assumption are not essential for heat spreading, the failure to account for boundary conditions may lead to an underestimation of the heat transfer rate in a non-laminated structure in cases where two-dimensional conduction is indeed significant; therefore, if a more accurate solution is available, it can be used as a substitute for local heat transfer relations. The maximization of this heat transfer metric within the interlayer space of the laminar composite, while simultaneously satisfying the requirements for minimum strength and encapsulation weight, is of primary importance for describing heterogeneous systems with a high concentration of impurity centers that disrupt the original direction of heat flux propagation.

Two additional scenarios involving the implementation of the multi-region fin model also rely on the foundational framework established in the primary research tables, though certain physical parameters require specific adjustments to maintain accuracy. In the first case, we consider a situation where both functional regions are cooled by the process of natural convection. This physical phenomenon is driven by density gradients within the cooling fluid, which are caused by temperature differences. Unlike forced convection, where fluid velocity is predetermined, natural convection requires the assignment of integral average convection coefficients separately for each domain. Specifically, unique coefficients must be designated for the heated region and for the lateral or fin region respectively.

The analytical solutions derived in the initial stages of this research remain structurally identical, but the application process changes: each distinct region must utilize its own specific convection coefficient and corresponding thermal resistance as

substitutions for the global parameters. It is important to acknowledge a fundamental limitation of this methodology. This approach cannot account for the high-precision variations of the locally changing convection coefficient within each specific domain. Attempting to include such level of detail would introduce an additional non-linear term into the governing differential equations. From a mathematical standpoint, this would necessitate coupling the heat transfer equations with the Navier-Stokes equations to solve for the exact behavior of the fluid flow. Such a complex coupling is typically avoided in simplified engineering models to maintain computational efficiency while providing a sufficiently accurate approximation for steady-state heat spreading analysis.

The second case of interest focuses on a scenario where the top surface of the heated section is treated as being perfectly well-insulated, while the lateral fin region continues to be cooled by a surrounding fluid. This configuration is particularly relevant for systems where heat is strictly directed through the internal core of the laminate toward the outer edges for dissipation. Under this condition, the heat transfer through the upper boundary of the heated zone is effectively zero. When a constant temperature boundary condition is applied at the source, the resulting temperature distribution within the heated domain is primarily governed by the interaction with the adjacent cooling region. The isolation of the top surface forces the thermal energy to propagate laterally through the high-conductivity core, emphasizing the role of the laminate as a heat-spreading manifold.

In both of these scenarios, the presence of localized foreign inclusions or impurities plays a significant role in disrupting the ideal heat flow. When such inclusions are present between the laminar layers, they act as thermal obstacles or enhancers depending on their specific material properties. In the case of low-conductivity inclusions, they create localized regions of high thermal resistance, which can lead to hotspots even when the global model predicts a relatively uniform temperature profile. Conversely, if the inclusions are metallic or carbon-based with high conductivity, they can assist the lateral spreading process, effectively acting as micro-fins within the structure.

The analysis of these systems also highlights the importance of understanding the transition between local and non-local heat propagation. In thin-film laminates, the boundaries between the layers are not just geometric dividers but functional interfaces that dictate the overall thermal efficiency. The multi-region fin model provides a robust framework for predicting how heat will move across these interfaces under varying external conditions. Whether the system is cooled by ambient air through natural convection or protected by high-performance insulation, the ability to separate the problem into distinct zones allows for a clearer understanding of the thermal bottlenecks.

Furthermore, these models are essential for the development of advanced thermal management systems where the weight and volume of the encapsulation must be minimized. By strategically adjusting the convection coefficients and accounting for the insulation of specific surfaces, engineers can optimize the thickness of the high-conductivity core to ensure that the maximum amount of heat is dissipated with the minimum amount of material. This is of particular importance in aerospace and mobile electronics applications, where every gram of weight and every millimeter of space is critical. The pseudo-steady-state analysis enabled by these models ensures that the thermal response of the laminate remains predictable even under fluctuating power loads or varying environmental temperatures.

Ultimately, the refinement of the two-region fin model to include specific convection scenarios and insulation conditions represents a significant step toward more realistic simulations of composite materials. It allows for the description of complex heterogeneous systems where the original direction of heat flux is intentionally manipulated to protect sensitive components or to maximize the efficiency of heat storage materials like salt hydrates and other phase change compounds.

Summary and conclusions.

This study demonstrates that multilayer laminate structures with high-conductivity internal cores are highly effective for managing localized heat fluxes. The proposed multi-region fin model provides a simplified yet accurate framework for

predicting temperature profiles and heat transfer rates across both Cartesian and cylindrical domains. By dividing the system into a heated region and a lateral fin region, the model effectively reduces complex two-dimensional conduction problems into manageable one-dimensional equations. It was found that laminates significantly outperform traditional homogeneous encapsulation materials, particularly when integrated with phase change materials for steady-state or pseudo-steady-state thermal storage applications.

The research concludes that achieving near-perfect temperature uniformity is possible through the strategic optimization of the high-conductivity core. Furthermore, the model remains robust under varying external conditions, such as natural convection or selective insulation. Maximizing the heat transfer metric while maintaining structural integrity and low weight is essential for modern thermal management. Ultimately, this analytical approach offers a reliable tool for designing advanced composite systems capable of dissipating heat in heterogeneous environments with high concentrations of local inclusions.

References:

1. Ida, K., Shi, Z., Sun, H. J., Inagaki, S., Kamiya, K., Rice, J. E., ... & Itoh, S. I. (2015). Towards an emerging understanding of non-locality phenomena and non-local transport. Nuclear Fusion, 55(1), 013022. DOI: 10.1088/0029-5515/55/1/013022

2. Moleiro, F., Soares, C. M., & Carrera, E. (2019). Three-dimensional exact hygro-thermo-elastic solutions for multilayered plates: Composite laminates, fibre metal laminates and sandwich plates. Composite Structures, 216, 260-278. DOI: 10.1016/j.compstruct.2019.02.071

Article sent: 19.12.2025